

Evaluating Quantitative Measures for Assessing Functional Similarity in Engineering Design

Ananya Nandy

Dept. of Mechanical Engineering
University of California, Berkeley
Berkeley, California
Email: ananyan@berkeley.edu

Andy Dong

School of Mechanical, Industrial,
and Manufacturing Engineering
Oregon State University
Corvallis, Oregon
Email: andy.dong@oregonstate.edu

Kosa Goucher-Lambert *

Dept. of Mechanical Engineering
University of California, Berkeley
Berkeley, California
Email: kosa@berkeley.edu

*Address all correspondence to this author.

ABSTRACT

The development of example-based design support tools, such as those used for design-by-analogy, relies heavily on the computation of similarity between designs. Various vector- and graph-based similarity measures operationalize different principles to assess the similarity of designs. Despite the availability of various types of similarity measures and the widespread adoption of some, these measures have not been tested for cross-measure agreement, especially in a design context. In this paper, several vector- and graph-based similarity measures are tested across two datasets of functional models of products to explore the ways in which they find functionally similar designs. The results show that the network-based measures fundamentally operationalize functional similarity in a different way than vector-based measures. Based upon the findings, we recommend a graph-based similarity measure such as NetSimile in the early stages of design when divergence is desirable and a vector-based measure such as cosine similarity in a period of convergence, when the scope of the desired function implementation is clearer.

1 INTRODUCTION

Quantifying the similarity or dissimilarity between multiple designs (e.g., commercial products) is essential for a variety of engineering design tasks. For example, design similarity is useful to identify products for benchmarking [1]. Furthermore, within a company, analysis of similarity can be used to evaluate the performance of product variants [2]. The concept of design similarity has also been used to determine patent infringement [3], as well as to investigate how product style changes over time [4].

In early-stage design, the concept of similarity has been found to be particularly important for design-by-analogy. As information across different domains of designs becomes increasingly accessible, it has become possible to leverage this data to provide sources of inspiration to designers. Design-by-analogy is a commonly used method to transfer knowledge from cross-domain sources and apply it to a target domain [5, 6]. When humans retrieve analogies on their own, they can have difficulty moving past surface-level similarities, often finding within-domain analogs that share readily-observed external attributes rather than underlying structural similarity. The

presentation of computationally-determined real-time analogical stimuli during early-stage design has been found to help designers produce novel outcomes [7]. Design cognition work has shown that different analogical ‘distances’ can impact a designer’s ideation processes, even on a neural level [8]. Therefore, it can be desirable to systematically control for the degree of similarity in design by analogy in order to leverage the effects of near vs. far analogs. In that case, it is critical to clearly define near and far through a measure of similarity.

Computational systems grant the opportunity to search for analogies at an underlying structural level within a larger space and automatically determine the relevant analogous design. Since these systems do not have to rely on surface-level similarities, they are able to retrieve more distant analogies based upon underlying functional patterns across domains [9]. Significant work has been conducted on analogy retrieval based on semantic representations of products such as design descriptions from a design problem solving session [7] or crowd-sourced design schema representations [9]. However, it can be advantageous to focus on functional analogies to further remove the possibility of fixation on surface similarities. To incorporate the advantages of functional representations, researchers have developed a method to retrieve analogies using a function-based approach on semantic data [10, 11].

Functional models offer an alternative to semantic descriptions, providing structured system or subsystem level representations that are useful for designers [12]. In addition, functional models have been useful to standardize design representations in methods such as bio-inspired design, where the source vocabulary is significantly different from that of the target [13]. A benefit of functional models is that they contain domain knowledge that can be mapped to a mathematical space, where a variety of measures are available to characterize the similarity between the designs.

The notion of functional similarity, critical to the comparison of designs, has not been tested extensively within research on design similarity — an opportunity we address here. It is possible that different measures can return drastically different analogs if the measures do not define similarity in the same way, analytically or conceptually, or if the measures define similarity in a way that is not actually congruent with the expectations of the designer. Other applications might also require similarity between designs to be defined in particular ways — different measures may

be appropriate for different contexts. This work empirically questions the meaning of *similarity* in engineering design by explicitly comparing multiple similarity measures and how they measure similarity across functional models. Specifically, we investigate the identification of similar functional models using vector space based methods, which are used frequently within the engineering design community [14]. In addition, we explore the possibility of representing functional models as networks by applying graph similarity measures. The work has direct implications for defining analogical distance for computational systems, but is relevant for any context where it is necessary to systematically determine the similarity between designs.

1.1 Similarity in Engineering Design

A variety of metrics have been used to define the similarity between designs. For instance, critical function chains have been extracted from functional models after which several matching metrics have been applied to determine similarity between the critical chains and find analogies [15]. Additionally, a vector-based similarity metric was developed to compare functional models from a product repository using customer needs. The metric represents designs as a linear combination of vector spaces, and has been shown to successfully find relevant analogies as demonstrated by its example application in finding analogies to drive the design of a new guitar pickup winder [14]. However, it can be difficult to ensure that domain specific knowledge, such as customer needs, is encoded within functional models. The metric has been used even without customer need weightings [16, 17], demonstrating an underlying assumption that notions of similarity will not significantly change when the additional information is omitted. Furthermore, a significant body of work has used natural language processing (NLP) in order to retrieve functionally similar designs from existing repositories (e.g., the U.S. patent database). Using NLP, designs are represented as vectors in a high-dimensional vector space and compared using the cosine similarity measure, which is typically used to compare texts of different lengths. [10, 11]. The use of latent semantic indexing and cosine similarity (NLP-based measurement) has been compared with the use of the previously described vector-based metric (functional model-based measurement) in the context of quantifying similarity between automatically generated concepts [18]. Finally, recent work has applied KL-divergence to determine similarity between designs in a way that embodies known principles of

similarity from cognitive science. This similarity metric represents knowledge about designs using probabilistic graphical models and maps design characteristics to performance measures before using KL-divergence to measure the similarity between design problems [19].

The engineering design community has also previously reviewed the use of similarity measures to compare designs at different stages of the design process. In a survey of similarity metrics used in engineering design, spatial function, vector space, edit distance, template model, and information theory approaches were evaluated holistically but qualitatively (i.e., no empirical results). It was determined that an edit distance or information theory approach should be most suitable to compute similarity between designs at the function structure stage of the design process [20]. Previous work has also investigated the dimensions of product similarity that are deemed the most important when humans make similarity judgments for finding useful analogies, with the results implying that function is more important than structure [21]. While various types of metrics have been used to try to find design similarity, there has been a lack of empirical testing on how the measures directly compare to each other and, therefore, no way to determine the appropriate situations to use each one.

Quantitative measures of similarity between different designs are important in applications such as design-by-analogy, given that prior research in engineering design reveals that analogies of varying distances can have different impacts on design outcomes. Analogical distance refers to how close the source design is to the target design, with analogies being divided into near-field and far-field analogies. Previous work often classifies within-domain systems as near analogies and out-of-domain systems as far analogies. Significant work has also been done to determine if the analogical distance affects the novelty or quality of ideas, since far-field ideas may have functional similarities that make them transferable. Studies on analogical distance have revealed contradictory results, pointing to the benefits of far-field analogies, but the problems when analogical distance is “too far” [10]. Thus, the results of this body of work indicate that the types of analogous designs presented affect their usefulness to designers. If the choice of similarity measure significantly influences what is considered functionally similar, then different similarity measures can offer new ways to find near-field and far-field analogies for design. More importantly, the outcome

may necessitate reassessment of the choice of metric used to find analogous designs.

1.2 Functional Modeling

Functional modeling is a group of methods by which a product can be decomposed into its key functions, providing an abstraction of the product that is useful for various stages of the design process. In early-stage design and concept generation, functional modeling can be used to decompose a complex design into simpler sub-designs by using a black box approach [22]. In addition, functional modeling provides a way to capture important knowledge about existing designs that is not captured in traditional documentation such as CAD models [23]. There are limitations to using functional models, especially in early-stage design, since existing functional models are typically created through reverse engineering. Additionally, there have been several approaches to functional modeling [22,24–27], which may cause the models to vary based on who created them. Mitigating this problem, the development of a functional basis has provided a common design language, allowing meaningful comparisons at the functional level [28].

The functional basis allows a product to be represented by labeling its functions and flows in pairs using a standardized vocabulary. There are three primary classes of flows (material, energy, and signal) and eight primary classes of functions (channel, support, connect, branch, provision, control magnitude, convert, and signal). These function and flow classes can have a further secondary and tertiary specification, maintaining flexibility in the level of abstraction at which a product can be modeled [23,28–30]. Once a product is modeled using these function-flow pairs, the model can be mapped to a variety of mathematical representations that can be used for further analysis. Specifically, the functional models can be mapped to a vector space or into a network / graph.

1.3 Measures for a Vector-Space Representation

The majority of prior work utilizing functional models has mapped the functional models into a vector space for similarity analysis. A functional model can be mapped to a vector space by building a binary vector from each of the function-flow pairs. For example, this binary mapping was used for the vector-based metric for similarity based on customer needs, as well as to investigate the effect of varying the level of abstraction on functional similarity [14,16]. A variation of the binary

mapping has been used to represent a higher level of abstraction by separately accounting for the existence of unique functions and unique flows, instead of counting unique function-flow pairs [17].

In the method developed by McAdams and Wood, the elements of each functional model mapped to a vector are weighted according to a customer needs rating. Then, product vectors are constructed into a product-function matrix, which is normalized for product complexity and customer enthusiasm rating, and re-normalized to unity. The inner product is then calculated between each product vector [14]. When the same process is followed without assigning weightings according to customer needs, the results are equivalent to applying the cosine similarity metric, which measures the cosine of the angle between the two non-zero vectors. The cosine similarity varies between zero and one, with one being perfect similarity. The cosine similarity has commonly been used in an engineering design context because of its applicability to any vector-based representation, including semantic representations of designs. However, in the absence of domain-specific weighting on specific functions (e.g. from customer needs), other metrics are available to quantify the similarity or distance between two binary vectors and may be applicable to compare functional models. Many metrics have been developed for this comparison according to different requirements, only some of which are investigated here as representatives from different classes of metrics [31].

1.4 Measures for a Network Representation

Networks mathematically represent the connections between entities. A network consists of vertices (or nodes) that are connected by edges. They have been widely used in applications such as social network analysis and have recently been utilized by the engineering design field. For example, recent work has used networks to find bridges between ideas from different domains using topic models [32] and to represent a conceptual design space for early-stage design [33]. In both cases, the networks are built from information in text documents and are not specific to functional representations. At a system level, networks have been used to represent complex systems and model system failure [34] as well as for the bio-inspired design of a power network [35]. They have also been applied to represent influential function models in a product architecture [36] and to investigate product transformation using graph edit distance on functional models [37].

The edit distance has also been previously noted as a relevant similarity measure for comparing function structures [20]. While networks are not currently widely used by the engineering design community to represent product function or functional models, using network-based measures to calculate functional similarity remains a promising direction that is further explored in this research.

Just like there are a variety of vector-based approaches to assess similarity, there are numerous network similarity measures that can capture the structure of a network. These can be divided into ones that require known node-correspondence — having a set or subset of matching nodes — and those that can have unknown node-correspondence [38]. In addition, network similarity measures can rely on network properties such as if the networks are undirected or directed (pointing only in one direction) as well as if they are unweighted or weighted (the edges have a positive continuous value) [39]. As such, functional models can be represented as networks from which similarity is calculated in several ways. Some representative measures are chosen here to illustrate the possible uses of network measures as measures of functional similarity.

2 RESEARCH METHODOLOGY

This paper investigates the impact of using different similarity measures on designs that may be similar at the functional level. First, the functional models were mapped to the desired mathematical space (a binary vector or a network) as shown in Figure 1. Then, a similarity matrix between all of the models in the repository was computed for each measure. The similarity matrices were all range normalized so that the similarity score would be between zero and one. The measures were quantitatively and qualitatively compared to gain insights regarding how each measure considers designs to be similar.

2.1 Functional Model Data

In order to investigate a variety of quantitative techniques for measuring design similarity, in this work we employed two separate datasets, each comprised of many functional models. The first dataset, focused on a within-domain application, contained 39 energy harvesting devices [17]. The second dataset, focused on cross-domain applications, included 61 consumer products (e.g., kitchen appliances, toys, power tools, etc.) [40]. The energy harvesting dataset was chosen because designs in this dataset had previously been grouped into similar categories of designs,

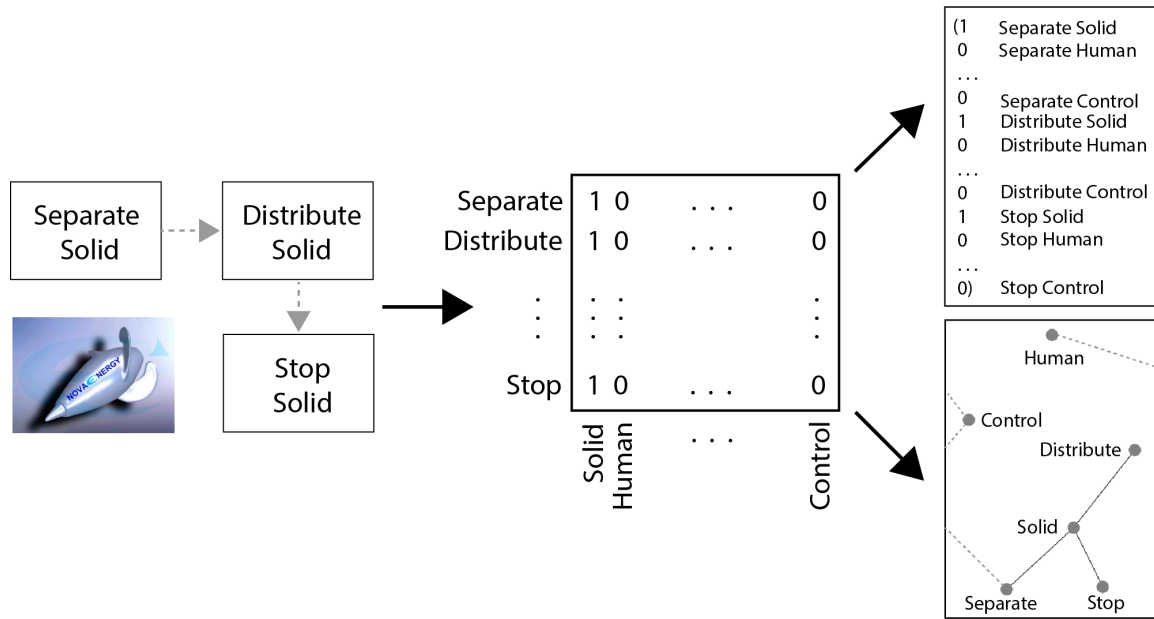


Fig. 1. The functional model data is in the form of a binary matrix connecting functions and flows. This matrix is flattened to a vector (top) or used as an adjacency matrix and represented as a network (bottom) as shown in this example with the Nova Energy Tuna Turbine. If the matrix contains a 1, an edge connects the function node to the flow node. If the matrix contains a 0, those nodes are not connected. Functions can only be connected directly to flows, not other functions, and flows can only be connected directly to functions, not other flows. The datasets include no information regarding the sequential ordering of function-flow pairs.

providing a reference for a human-based assessment of similarity. Designs in both datasets had already been represented as functional models following rigorous protocols, thereby ensuring consistency in the functional models within a dataset.

Analysis techniques were the same for each dataset. For both datasets, functions were specified to the secondary level while flows were (sometimes) specified to the tertiary level. For example, the flow *mechanical energy* was clarified further to be *rotational* or *translational*. The functional models did not include information about the sequencing of function-flow pairs in the system, the repetition of any functions/flows, or the relative importance of any functions/flows. While both datasets utilized the functional basis to specify product function, these functions were encoded slightly differently, and such, the datasets are not directly compared to each other.

Energy harvesters. Since the set of energy harvesting devices is inherently similar by technology, the similarity measures were expected to find systems that share a similar working principle to be “near” in that dataset. The systems included energy harvesters that implement different technologies to convert energy from one form to another and ranged from prototypes to com-

mercial products. The energy harvesting functional models were categorized into technological sub-domains as follows:

- 9 inductive vibration harvesters
- 6 piezoelectric vibration harvesters
- 6 wind harvesters
- 3 ocean-current/wave harvesters
- 6 solar harvesters
- 5 thermal harvesters
- 4 hybrid harvesters

Details about the categorization of the energy harvesting devices can be found in Appendix A.

Consumer products. The consumer product dataset was analyzed because it encompassed a wider range of designs that were not necessarily from the same technological domain. These products were not placed into sub-categories. The consumer product functional models were specified using a vocabulary of 20 functions and 18 flows, compared to the energy harvesting devices, which were described using a vocabulary of 21 functions and 16 flows (a list can be found in Appendix B). The consumer product dataset was modified to match the level of abstraction of the energy harvesting device dataset as much as possible, based on the functional basis vocabulary. However, since the product function was determined separately and in a different context in each case (fewer function-flow combinations were explicitly specified in the consumer product dataset), there may be differences in the encoding of function between the two datasets. On average, a device in the energy harvesters dataset contained 13 unique functions, 8 unique flows, and 107 total function-flow pairs. On the other hand, a device in the consumer products dataset contained 10 unique functions, 7 unique flows, and 16 total function-flow pairs. Further details about the numbers of functions and flows can be found in Appendix A.

2.2 Measures of Similarity Using Vectors

The functional models were mapped to a vector space by building a binary vector from the existence of function-flow pairs in the system. Therefore, for n functions and m flows, each functional model was represented by a vector of zeros and ones of length $n \times m$. These vectors were

then used for any similarity computation. The similarity measures were chosen only if they were applicable to binary data. The similarity measures do not always satisfy the specific definition of a *metric* and therefore are not referred to as such. In addition, there was an effort to include measures that have been previously used in or have the potential to be used in the engineering design field. It should be noted that some measures are referred to (or calculated) as distances and dissimilarities. These were always converted to measures of similarity before comparison. The vector-based similarity measures explored in this work are described in more detail below.

Simple matching coefficient (SMC). The Hamming distance is the number of differences in corresponding positions of two binary vectors. Eq. 1a shows that the formula for Hamming distance is

$$\text{Hamming distance} = \sum |x_1 - x_2|, \quad (1a)$$

where x_1 and x_2 are the two binary vectors being compared. The measure is often divided by n (vector length) in computational packages to obtain a proportion. This proportion can then be converted to the simple matching coefficient (*SMC*) as

$$\text{SMC} = 1 - \frac{\sum |x_1 - x_2|}{n}. \quad (1b)$$

The SMC can only be used on binary data and is useful if the features are symmetric. This means that the absence or presence of the feature carries equal information.

Jaccard similarity coefficient. The Jaccard similarity coefficient and the SMC are close in their comparison of binary vectors. Eq. 2 shows that the formulation of the Jaccard similarity coefficient is

$$\text{Jaccard similarity} = \frac{|x_1 \cap x_2|}{|x_1 \cup x_2|}, \quad (2)$$

where x_1 when x_2 are the two binary vectors being compared. Unlike the SMC, however, the Jaccard similarity coefficient excludes any features that are not present in either vector. Therefore, it only accounts for mutually present matches between the vectors. The Jaccard similarity as defined above can be used on binary data and modifications allow the measure to be used with weights or probability distributions.

Cosine similarity. The cosine similarity determines similarity based on the the angle between two vectors in a vector space. It is

$$\text{Cosine similarity} = \frac{x_1 \cdot x_2}{\|x_1\| \|x_2\|}, \quad (3a)$$

where x_1 and x_2 are the two vectors being compared. The quantitative similarity metric developed by McAdams reduces to this measure when information about customer needs is not available. When used on binary data, the equation can be rewritten as

$$\text{Cosine similarity} = \frac{|x_1 \cap x_2|}{\|x_1\| \|x_2\|}. \quad (3b)$$

The numerator is the same as in the Jaccard similarity coefficient from Eq. 2. The cosine similarity can be used on binary data but does not need the data to be binary. It is commonly used in the context of comparing text documents of different lengths since it compares the orientation of two vectors in a high-dimensional abstract space.

2.3 Measures of Similarity Using Networks

To represent a functional model as a network, the functions and flows of each product were first mapped to a binary matrix. Each function and flow was represented as a node and the edges were determined by the values in the matrix. Edges between functions and flows existed only if the binary matrix had a 1 in the corresponding row and column. The network comparison measures were chosen based on prior work that compares these types of measures using random graph

models and different (i.e. not design-related) real world networks in order to make recommendations related to their application based on local and global structure [39, 41]. The representative measures were chosen so that they would be applicable to undirected and unweighted networks because the functional models do not contain information about the direction of connections between different functions or about relative importance of functions (note that weighting can be handled by the chosen measures but different measures might be needed for directed networks). In addition, single feature-based approaches for network comparisons (clustering coefficient, centrality, etc.) were not considered. The graph similarity measures were formulated as distances and then converted to similarity for comparison. The networks were visualized and analyzed using the NetworkX [42] and NetComp [41] libraries in Python.

NetSimile. NetSimile finds a graph’s “signature” vector based on features of its nodes [43]. The features that are included are the node degree, the node clustering coefficient, average degree of node neighbors, average clustering coefficient of node neighbors, number of edges in the node’s egonet, number of neighbors in the node’s egonet, and the number of outgoing edges from the node’s egonet. A node’s egonet is an induced subgraph of the neighbors centered around a node within a certain radius. For example, in this case, an egonet would consist of a function and any connected flows. The features are aggregated across nodes. Then, the Canberra distance,

$$Canberra\ distance = \sum \frac{|x_1 - x_2|}{|x_1| + |x_2|}, \quad (4)$$

is calculated between two feature vectors x_1 and x_2 where each element of a vector is the mean, median, standard deviation, skew, or kurtosis obtained from feature aggregation.

Spectral distance. Spectral distances are based on the eigenvalues of a matrix. In this case, the spectral distance is defined as

$$Spectral\ distance = \|\lambda_{\mathcal{L}_1} - \lambda_{\mathcal{L}_2}\|, \quad (5a)$$

where $\lambda_{\mathcal{L}_1}$ and $\lambda_{\mathcal{L}_2}$ are the eigenvalues of the Laplacian matrices ($\mathcal{L}_1, \mathcal{L}_2$). The Laplacian matrix is

$$\mathcal{L}_i = D_i - A_i. \quad (5b)$$

In addition to using the adjacency matrices (A_1, A_2) which indicate whether nodes, functions and flows, are connected, the spectral distance accounts for the degree matrices (D_1, D_2) through the Laplacian. The degree matrix is a diagonal matrix that indicates how many other nodes each node is connected to.

When the Laplacian matrix is normalized, the spectral distance can be used to compare graphs of different sizes. In addition, it does not require the nodes of the two graphs to be the same. When computing a spectral distance, the number of eigenvalues that are considered can be adjusted, allowing flexibility in considering community structure (fewer eigenvalues) or including local structure (more eigenvalues). Comparisons of several types of real world networks finds that spectral distance is a reliable measure for different applications [41].

DeltaCon distance. The DeltaCon distance is a graph comparison measure intended to account for the similarities in connectivity between two graphs. The pairwise node affinities are calculated for each graph and compared to each other. The node affinities are calculated using a concept called fast belief propagation (FBP), an approximation of the loopy belief propagation algorithm. This is a message-passing algorithm often used on graphs in computer science [44]. The FBP matrix is

$$S = [I + \epsilon^2 D - \epsilon A]^{-1}, \quad (6a)$$

where ϵ is

$$\epsilon = \frac{1}{1 + d_{max}}. \quad (6b)$$

ϵ is the constant that accounts for the influence of neighboring nodes and is computed using the maximum value in the degree matrix (d_{max}). The FBP matrix can also be written as

$$S \approx I + \epsilon A + \epsilon^2(A^2 - D) + \dots, \quad (6c)$$

demonstrating how it incorporates information about neighboring nodes using weighting. The final distance is then

$$\text{DeltaCon distance} = \sum |\sqrt{S_1} - \sqrt{S_2}|. \quad (6d)$$

Like the spectral distance, the DeltaCon distance uses both the adjacency matrix (A) and the degree matrix (D). Fast belief propagation is intended to track the spread of information through a graph, making the DeltaCon method good for local and global structure [41].

3 RESULTS

The vector and network-based similarity measures outlined in Section 2 were used to find the similarity between the functional models of all pairs of devices in the energy harvesting data and in the consumer product data. The results were stored in similarity matrices and then analyzed with the objective of determining if the similarity measures return consistent results across functional models and are measuring the same construct of similarity.

3.1 Overall Comparison of Similarity Measures

The similarity matrices, which are pairwise comparisons of the functional models as evaluated by each similarity measure, were plotted as a distribution of scores. Comparing the distribution of scores provides insight into if the measures are measuring comparable concepts of design similarity at the functional level. The kernel density estimate of each similarity measure is shown in Figure 2 for the energy harvesting devices and Figure 3 for the consumer products.

The mean and Pearson's coefficient of skewness of the distributions were calculated for each

measure as shown in Table 1 and Table 2. A large negative coefficient of skewness indicates that the mass of the distribution is concentrated on the right (higher similarity), while a large positive coefficient of skewness indicates that the mass of the distribution is concentrated on the left (lower similarity).

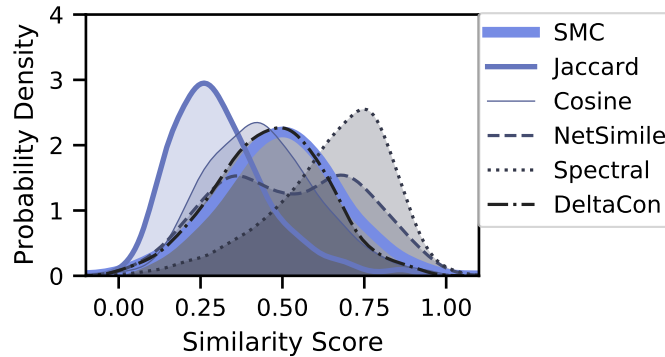


Fig. 2. Distribution of normalized similarity measures for energy harvesting devices.

Table 1. Mean similarity scores and coefficient of skewness of all energy harvesting devices. Shaded rows indicate measures that have highly skewed distributions.

Measure	Mean Similarity Score	Skew
SMC	0.49	-0.07
Jaccard	0.31	0.97
Cosine	0.44	0.35
NetSimile	0.52	-0.03
Spectral	0.65	-0.81
DeltaCon	0.47	-0.06

The mean of the spectral similarity was the highest at 0.65 and 0.63 while the mean of the Jaccard similarity was the lowest at 0.31 and 0.21. The distributions of similarity scores from these two measures were highly skewed for both datasets. However, they were skewed in opposite directions, which was unexpected. The Jaccard similarity distribution was concentrated towards lower similarity, while the spectral similarity was concentrated towards higher similarity. The distribution of the cosine similarity was moderately skewed. All other measures had distributions that had a low skew. A set of pairwise Mann-Whitney U tests between the two distributions reveals that

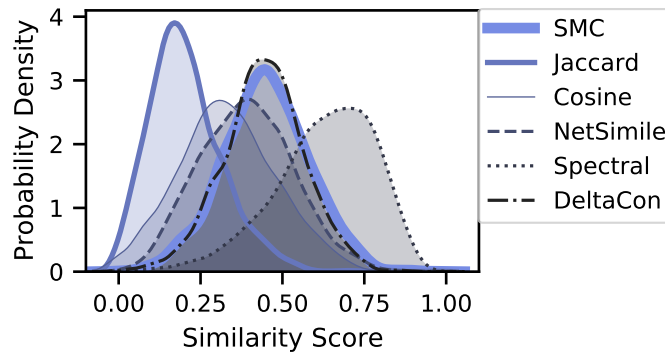


Fig. 3. Distribution of normalized similarity measures for consumer products.

Table 2. Mean similarity scores and coefficient of skewness of all consumer products. Shaded rows indicate measures that have highly skewed distributions.

Measure	Mean Similarity Score	Skew
SMC	0.45	0.003
Jaccard	0.21	1.00
Cosine	0.33	0.26
NetSimile	0.39	0.15
Spectral	0.63	-0.72
DeltaCon	0.44	-0.09

the null hypothesis (i.e., that the populations are equal), is rejected ($p \leq 0.05$). This indicates a significant difference between the measures in relation to their computation of similarity between designs in the two datasets. For example, one clear difference is evident from the kernel density estimate of the NetSimile measure, which demonstrates two peaks for the energy harvesting dataset but only one peak for the consumer products dataset.

Next, the similarity matrices were used to determine if the results returned by each similarity measure were distinct. Even if the distributions differed, it was possible that the rankings of each product compared to each other product would not significantly differ among measures. For each product, every other product was ranked in order of its similarity to the initial design (tied rankings were included). The purpose of examining the rankings was to consider the possibility that even if the *value* of similarity between two measures were different, the relative *order* of systems re-

turned may not differ much. These rankings were then analyzed using the Kendall rank correlation coefficient (Kendall's τ) to obtain a pairwise comparison between the methods.

Due to existence of a distribution of rank coefficients that depended on the initial system and because of the small sample size, bootstrapping was used to find the 95% confidence interval for the pairwise rank coefficient, as shown in Figure 4 and Figure 5. A positive rank correlation coefficient close to one indicates that the two measures being compared return rankings that are similar (i.e., they find the same types of functional models similar).

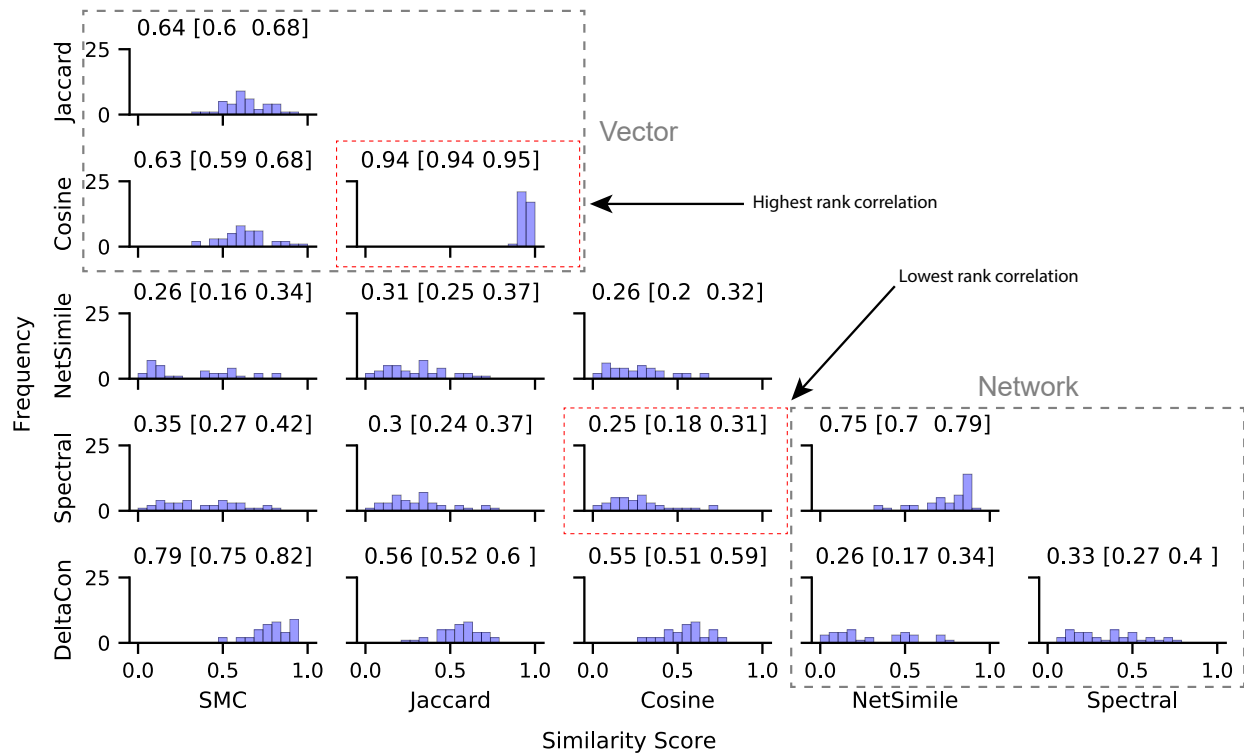


Fig. 4. Kendall rank correlation coefficients between similarity measures are shown, using each of the 39 energy harvesting devices as the “target” design. The mean and 95% confidence interval from bootstrapping (n=500) is shown for each pair of measures.

Despite there being a distribution of rank correlation coefficients, the rank correlation analysis revealed only a moderate correlation between most similarity measures. However, the Jaccard and cosine similarity measures were highly correlated within a relatively narrow interval. In addition, the spectral similarity measure showed a very weak correlation with all vector measures and only showed a higher correlation with NetSimile, another network measure. SMC, a vector measure, and DeltaCon, a network measure showed a moderately high correlation despite significant

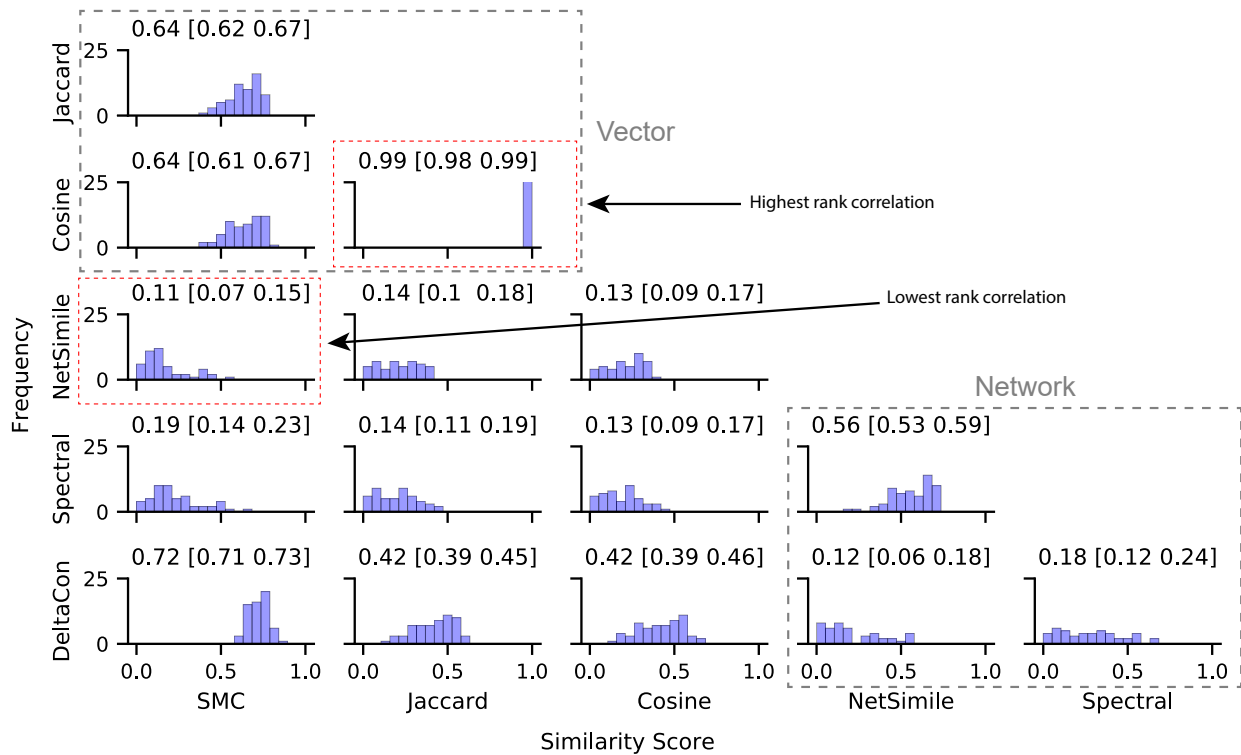


Fig. 5. Kendall rank correlation coefficients between similarity measures are shown, using each of the 61 consumer products as the “target” design. The mean and 95% confidence interval from bootstrapping (n=500) is shown for each pair of measures.

differences in their mathematical formulation. These results were consistent across both datasets.

3.2 Comparison of Measures within Categories

The energy harvester functional models shared a common intended purpose, with each device further labeled as a specific type of energy harvesting device (wind, solar, etc.). It was expected that devices within these categories have similar working principles and hence should be similar. The categorizations for each device can be found in Appendix A. The mean similarity was calculated for the systems within these predefined energy categories as shown in Table 3. Highlighted cells indicate category-level mean similarity scores that are not greater than or equal to the overall mean similarity score.

The within-category similarity was generally higher than the mean similarity of all energy categories, although statistical significance was not determined due to small sample size. Hybrid systems, which were predefined to contain multiple energy categories, had a lower within-category

Table 3. Mean similarity scores of energy harvesting devices grouped by category. Shaded cells indicate within-category means that are lower than the overall mean for the similarity measure.

Measure	Category Mean Similarity Score							Mean Similarity Score
	<i>Inductive</i> (n=9)	<i>Piezoelectric</i> (n=6)	<i>Wind</i> (n=6)	<i>Wave</i> (n=3)	<i>Solar</i> (n=6)	<i>Thermal</i> (n=5)	<i>Hybrid</i> (n=4)	
SMC	0.61	0.64	0.75	0.67	0.54	0.64	0.45	0.49
Jaccard	0.41	0.31	0.58	0.44	0.40	0.52	0.38	0.31
Cosine	0.55	0.44	0.72	0.59	0.55	0.64	0.52	0.44
NetSimile	0.59	0.43	0.57	0.44	0.56	0.56	0.53	0.52
Spectral	0.71	0.70	0.75	0.72	0.61	0.68	0.54	0.65
DeltaCon	0.60	0.67	0.67	0.59	0.54	0.58	0.39	0.47

similarity in half of the measures. However, piezoelectric devices had a lower within-category similarity using the Jaccard, cosine, and NetSimile measures while the hybrid devices did not. Other exceptions included solar devices, which had a lower within-category similarity using the spectral measure and wave devices, which had a lower within-category similarity using the NetSimile measure.

Given the assumption that within-category devices should reasonably share the same working principle as well as higher similarities for within-category devices, it was expected that the device pairs that were considered the “most similar” would be devices of the same category. Table 4 shows the pairs of energy harvesting systems that were considered the most similar by each measure. The systems are color-coded by category (found in Appendix A).

This was true in almost every case, where the pair of most similar systems was a set of either thermal, wind, or solar harvesters. However, the spectral measure returned different pairs than any of the other measures, finding devices from different categories to be the most similar. In addition, the spectral measure returned groups that had a similarity score of 1 (perfect similarity), despite containing different devices. Even for the other measures, there was no agreement which specific devices were “most similar” in absolute terms (e.g. which was more similar, the pair of wind harvesters or the pair of thermal harvesters?).

Table 5 shows the pairs of consumer products that were considered the most similar by each

Table 4. Pairs of devices with the highest similarity score (thermal, wind, solar, wave, inductive, and piezoelectric devices). For the spectral measure, there was a group of three devices that had the highest similarity.

Measure	Systems	Similarity
SMC	Micropelt STM-PEM Micropelt TE-power Ring	0.95
Jaccard	Four Seasons Enviro-Energies Tracking System Solar Heat Engine w/ Mirrors	0.88
Cosine	Four Seasons Enviro-Energies Tracking System Solar Heat Engine w/ Mirrors	0.93
NetSimile	Nova Energy Tuna Turbine WindTamer U Texas Prototype Heel-impact Shoe Harvester	1
Spectral	Wing Wave Generator Michigan U Piezo Flag Nova Energy Tuna Turbine WindTamer U Texas Prototype Heel-impact Shoe Harvester Columbia Power Manta Buoy Micropelt STM-PEM Enocean Eco 100	1
DeltaCon	Micropelt STM-PEM Micropelt TE-power Ring	0.91

measure. The results on this dataset showed much more agreement. This time, a coffee maker and iced tea maker were considered to be the most similar by every measure except for the spectral measure. The agreement indicates less uncertainty around what products are functionally similar in the consumer products dataset compared to the energy harvesting dataset.

The similarity matrices were then subjected to hierarchical clustering to determine if the different high-level categories could be recovered from the lower-level functional attributes using any of the measures. Figure 6 shows an example of hierarchical clustering using the Jaccard measure.

Table 5. Pairs of products with the highest similarity score

Measure	Systems	Similarity
SMC	Mr. Coffee Coffee Maker-RBS West Bend Iced Tea Maker-RBS	0.97
Jaccard	Mr. Coffee Coffee Maker-RBS West Bend Iced Tea Maker-RBS	0.96
Cosine	Mr. Coffee Coffee Maker-RBS West Bend Iced Tea Maker-RBS	0.98
NetSimile	Mr. Coffee Coffee Maker-RBS West Bend Iced Tea Maker-RBS	0.94
Spectral	Presto Popcorn Popper Horseman Swimming Toy	0.95
DeltaCon	Mr. Coffee Coffee Maker-RBS West Bend Iced Tea Maker-RBS	0.98

Although there are several methods available to choose the number of clusters, seven clusters were chosen in order to ascertain the measures' ability to cluster the designs into the general categories of energy harvesting devices based on the semantic label (wind, wave, solar, etc.). None of the measures were able to recover these category groupings perfectly.

To test the differences in groupings across measure, Table 6 shows the largest percentage of within-category devices that were grouped together. For example, for the Jaccard and cosine measures, all of the wind harvesters were grouped together, while 4/5 of the thermal harvesters were grouped together. On average, the vector measures grouped "more similarly" to the higher-level categories, having a higher mean percentage of within-category devices in a cluster. The network measures, especially the NetSimile and spectral measures, tended to not group designs based on the higher-level categories. This indicates a clear difference between the definition of similarity at a higher-level vs. lower-level description of function using the network measures.

4 DISCUSSION

This work shows that the choice of similarity measure changes empirical findings on design similarity. Calculations of design similarity are therefore sensitive to *different concepts of similarity* rather than a normative conception of similarity. Based on previous qualitative analysis on the

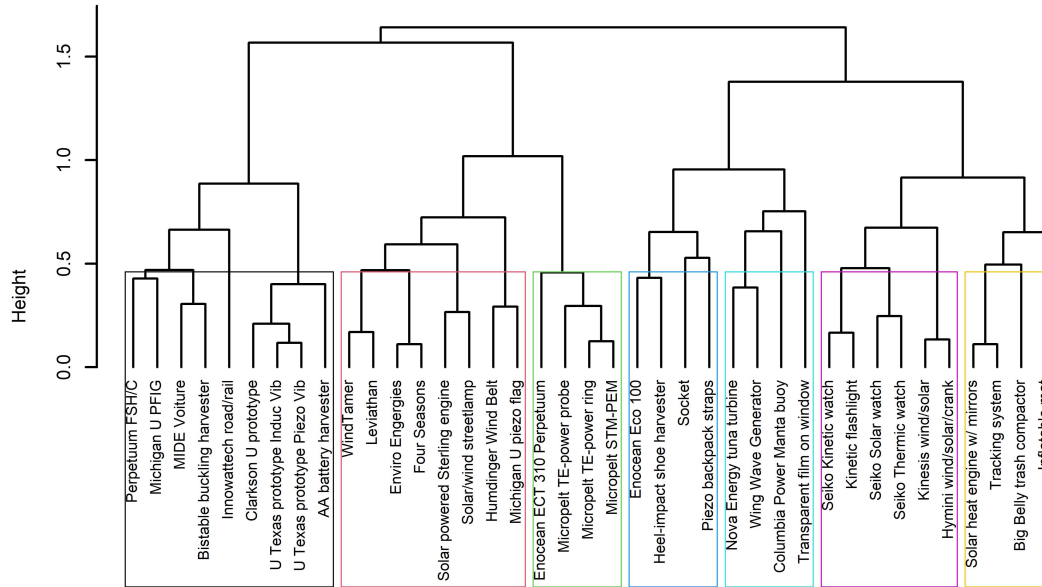


Fig. 6. Hierarchical clustering using the Jaccard measure on the energy harvesting dataset. Seven clusters are selected corresponding with the seven predetermined categories.

Table 6. The largest percentage of energy harvesters within a certain category clustered together by each measure, using hierarchical clustering with seven clusters. Shaded cells indicate the measures with the lowest percentage within a category.

Measure	Category							Mean
	<i>Inductive</i>	<i>Piezoelectric</i>	<i>Wind</i>	<i>Wave</i>	<i>Solar</i>	<i>Thermal</i>	<i>Hybrid</i>	
	(n=9)	(n=6)	(n=6)	(n=3)	(n=6)	(n=5)	(n=4)	
SMC	56	67	67	100	50	80	50	67
Jaccard	56	67	100	100	67	80	50	74
Cosine	56	67	100	100	50	80	100	79
NetSimile	44	33	50	67	33	40	50	45
Spectral	33	33	50	67	33	60	25	43
DeltaCon	56	67	67	100	50	80	50	67

energy harvesting device data used in this study, it was determined that all of the energy harvesting devices have a similar function structure in general, but differ in some supplemental functions and flows. This overarching structural similarity was not captured in the quantitative metric originally used to compare the devices [17]. Even here, the different measures show different spreads of

similarity values as shown in Figure 2. The spectral distance has been found to work well to distinguish designs when networks have very similar degree matrices but not the same specific functions. In this context, it can be expected that the degree distributions are very similar around common functions, such as *convert* (a key function for energy harvesting devices since they are all converting some input to a form of usable energy flow). On the other hand, measures such as the Jaccard similarity find the complete opposite, indicating low similarity among the set of devices. Based on this, it seemed as if the spectral measure would be able to reflect a domain level similarity among the energy harvesting systems through its skew towards higher similarity scores. Given that the results skew high even with a dataset of products that are not in the same technological domain, this is not likely the case. However, the spectral measure does appear to capture something that is not captured by the other measures — a potentially varying notion of similarity. In an attempt to understand what exactly it is that the network measures are capturing, the designs were changed from their original form to observe the resulting impact on similarity.

4.1 Similarity Measures Applied to Perturbed Data

The measures were examined closely to investigate specific aspects of functional similarity that are captured by different measures. For instance, is similarity determined by individual function-flow pairs, patterns of function-flow pairs, or an overall structure of connections between functions and flows? Two additional analyses were conducted: one to examine how a specific function and all of its connected flows affected the similarity and one to examine how switching the flows associated with function-flow pairs affected the similarity. Insights gained from the additional analyses inform when network measures may and may not be suitable in the context of engineering design.

4.1.1 Search for specific functions

Understanding the level at which specific function-flow pairs influence the similarity might be desirable in order to find similar designs using only parts of a functional model. Parts of the functional model can be targeted by focusing on how the measures operate on subgraphs of the functional model. The subgraph considered here is one function and all of the flows that are immediately connected to it (also called the function node's egonet) which we call the function subgraph. The similarity was calculated between the function subgraph of one product and the

corresponding function subgraph of all other products (using the energy harvesting dataset). Then, the similarity was calculated between the function subgraph of one product and the full graphs of all products. The rankings were compared using Kendall rank correlation coefficients. Figure 7 shows the process as well as examples of rank correlation results.

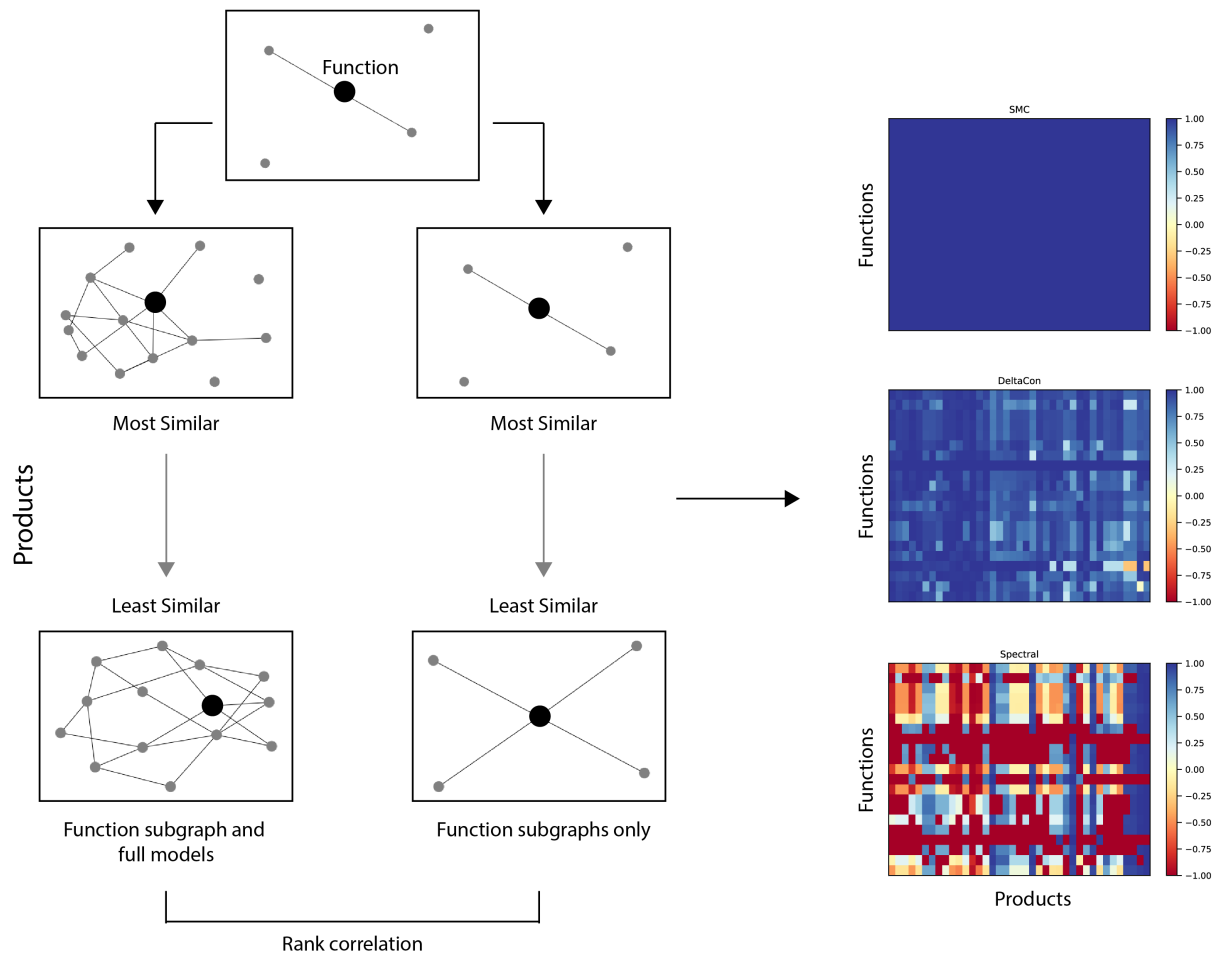


Fig. 7. Similarity rankings for a function subgraph in one product and the full functional model of all products are correlated with similarity rankings for a function subgraph in one product and the corresponding function subgraphs of all products. A shaded correlation matrix across all functions and products shows that vector-based measures such as SMC find rankings to be highly correlated while network-based measures such as the spectral distance have much more variation, indicating a significant difference in the importance of specific function subgraphs for determining functional similarity.

For the SMC measure, using a subgraph as an “input” and considering similarity with the full models returns the same rankings as considering similarity with the corresponding function sub-

graphs. Therefore, searching for a subgraph within a full model is likely to return consistent results with just comparing the subgraphs using this measure. The consistently high rank correlation is not present with network-based measures such as the spectral measure, with the rank correlation depending on what product or function is the “input.” This variation indicates that these measures are likely to be less useful when searching for specific subgraphs within a larger model, as the similarity is not captured at the level of specific subgraphs. There are indications from this analysis that the network-based measures do not place importance on what the specific functions are within the functional model.

4.1.2 Determination of specific flows

One limitation of using functional models for design-by-analogy in the early design process is that determining specific functions and flows indicates a higher level of detail than may be desired at that stage. However, measures that place less importance on the specific functions/flows and more on the connections between different functions/flows can be useful in overcoming this limitation. To illustrate this idea, the functional model was perturbed by swapping the columns of the encoding binary matrix. In the network view, swapping the columns means changing the name of the network’s nodes. In a design context, changing the node names might represent, for example, changing the functional model of a motor to that of a generator - instead of converting electrical energy to mechanical energy, doing the reverse. In this case, the perturbation is a direct swap of flow nodes with no consideration of whether the perturbed model is a realistic one or if unrealistic function-flow pairs are created. However, swapping the nodes can show whether the specification of individual functions or flows affects the computational determination of similarity. All unique combinations of flow nodes were swapped and then the similarity between the perturbed model and the original model was calculated. These similarity scores were then averaged to obtain a single value for each energy harvesting device.

Figure 8 indicates that two of the network based measures (NetSimile and spectral) are less affected by the specific flows — demonstrated by the higher similarity values. This indicates that functional models with relevant connections will be considered similar even if the flows are transient, demonstrating the ability of the two measures to capture the overall structure of the

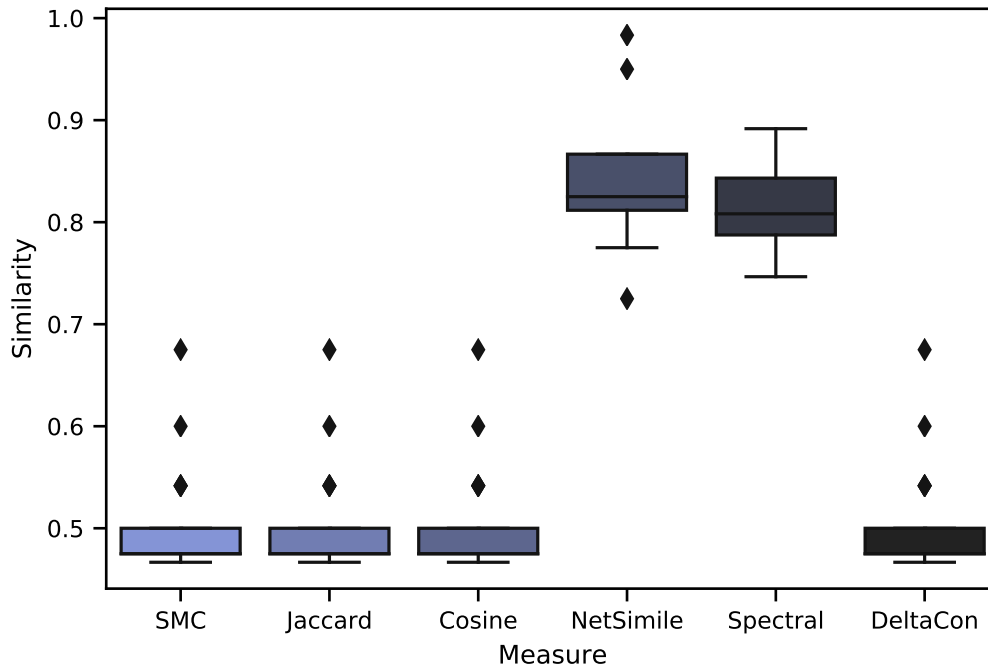


Fig. 8. Similarity scores across measures for energy harvester designs perturbed by swapping pairs of “flow” nodes.

functional model without prioritizing the details (i.e. specific functions and flows).

4.2 Implications for Design

Being able to compare the similarity between designs remains critical for leveraging computation in engineering design. Examining the impact of using a particular measure across the categorized energy harvesting device dataset provided insight into what should be considered when computing design similarity from a functional perspective. For example, the network measures do not necessarily scale similarity in the same way as more simple measures, demonstrated by the variation in distributions of similarity scores. Additionally, network measures do not appear to categorize the energy harvesting devices into the human-determined technological categories as well as the vector-based methods that match different functions and flows, as shown by the clustering results. Prior work from Weaver et al. qualitatively indicated that the energy harvesting devices generally have a similar function structure, but differ in some supplemental functions and flows. If there is an overarching structural similarity across the devices, measures should also find

cross-category devices to have high similarity [17]. Measures such as the cosine and Jaccard similarity, which focus on individual functions (e.g. matching the existence of “liquid” across wave generators) may not be well-suited for this purpose. However, it is possible that network measures, which assess similarity more holistically, are a viable alternative. The network measures used in this work do seem to capture aspects of functional model structure as represented by the connections between functions and flows rather than the specific functions and flows themselves. This property is demonstrated by their robustness to the swapping of specific nodes. Therefore, even in the functional model’s simple undirected form, there is a possibility to use these network measures when searching for unintuitively similar ways in which a design might work or to find cross-category designs. Table 7 summarizes these considerations, based on the analyses conducted in this work.

Table 7. Qualitative comparison of the characteristics of each measure. Score Distribution is considered across both datasets while all others are considered based on analysis using the energy harvesting device dataset. For Within-category Grouping and Subsystem Match the measures are qualitatively ranked from 1 (Best) to 6 (Worst).

Measure	Score Distribution	Within-category Grouping	Robust to Perturbation	Subsystem Match
SMC	No skew	3	No	1
Jaccard	Low skew	2	No	3
Cosine	Low skew (slight)	1 (Best)	No	1
NetSimile	Inconsistent	5	Yes	5
Spectral	High skew	6 (Worst)	Yes	6
DeltaCon	No skew	3	No	4

4.2.1 *What to consider when choosing a similarity measure*

In this study, vector and network measures are empirically compared across two functional model datasets. The results reveal that network measures that have not been widely applied to design similarity, such as the spectral distance and NetSimile, do indeed return “similar” designs in a way that is different from other measures. This is indicated by the low rank correlation coefficients between network measures and all the vector-based measures. This demonstrates how differently network-based measures are impacted by perturbations and subgraph-level searching.

It is possible that the abstraction level of the design representation plays a significant role in these differences.

Prior work has used pruning rules to distill detailed functional representations into those that only include the core functions and flows (a higher abstraction) in an attempt to ensure that the functional similarity determined by vector-based measures is more meaningful [16, 45]. It is useful to conduct this type of pruning before using a measure such as the Jaccard similarity because the presence of a core function or flow between two designs will highly influence the similarity score. For example, based on how the SMC measure works on functional models, if two purely mechanical devices are both missing the “electrical energy” flow, their calculated similarity would increase. Using the Jaccard measure to compare the same two devices would not capture this implied similarity. However, reducing the information in the functional model might not be necessary when using a network measure since a core flow node might be connected to many function nodes (or vice versa) and this is directly incorporated into the similarity calculation. Specific comparisons at the individual function level play a smaller role in the network-based NetSimile and spectral distances. For example, in NetSimile the individual functions are not directly compared across devices, but are aggregated and then compared. Therefore, the comparison is at a higher-level.

Based on the ways in which each measure operates to compare functional representations, it is likely that both the level of complexity of designs and information contained in the functional model influence which measure should be used in a design context. In this case, we refer to increasing complexity as having an increasing number of function-flow pairs. The Jaccard similarity might suffer when comparing designs of varying complexity, as designs with high and low numbers of function-flow pairs inherently decrease in similarity score due to the mismatch (again, pruning may help here). Cosine similarity might also be affected by this difference in complexity, but one benefit of its formulation is that it can be used even if the functional structures are not represented in a binary way (i.e. the existence or absence of functions) and more information is included. The SMC is specifically affected when comparing two low complexity designs as the similarity might be artificially inflated by many missing functions, especially if the comparison is being made in a global set of devices of varying complexity. In many ways, the DeltaCon network measure is similar to

SMC, as it requires the functional structures to match in their node labels (functions/flows) before comparison. However, like cosine similarity and all of the other network measures, it carries the benefit of being flexible to utilizing additional information such as weighting. Finally, a network measure like NetSimile is specifically designed to handle comparisons of graphs of different sizes, making it more suitable for comparing designs of varying complexity.

4.2.2 Using similarity measures to find “near” or “far” functional analogies

Finally, if analogical stimuli are provided to designers computationally, the choice of similarity measure to retrieve the “right” stimulus becomes important. Work in analogical design has implied that stimuli from a “sweet spot” between near-field and far-field help designers in the design process, but has also noted that the meaning of near and far varies across the literature [10]. The results of this work indicate that the choice of similarity measure impacts what types of systems are considered functionally similar (and consequently, what might be returned as near or far). For instance, measures that are better able to capture structural similarity might return, as a near example, a result that another measure (or a human designer) might consider a far example.

More broadly, some measures might be better suited to either design exploitation or design exploration. A higher-level notion of similarity can be useful to provide unintuitive but similar examples that aid divergence during design exploration. However, it may not be useful when designers want to transfer aspects of an existing design that matches their needs to a new design. In the latter case, which can be important when converging on an idea later in the design process, it might be better to have a measure that returns designs that are more similar (e.g. for energy harvesters, a within-category device). Therefore, these results can lead to a better understanding of how specific similarity measures can be leveraged for specific purposes within engineering design.

For near (within domain) designs, the choice of similarity measure is not as critical — a majority of the measures tend to result in the same sets of systems returned as the most “similar.” However, if the functional model is represented as a network, very different results can be found using the spectral distance. Therefore, a network measure like the spectral measure may not perform particularly well to exploit already refined target designs, but could be more useful than vector-based methods for design exploration.

4.2.3 Recommendations and limitations

There are some limitations to this study. Mapping the functional models with no functions repeated, no information about sequential order of functions, or no weighting of importance, is unlikely to work for a very complex system that has most or all of the functions from the functional basis. A more complex system can be more easily be mapped to a network as demonstrated in previous work if more detail about function repetition or importance is available [34]. Even at the current level of detail, however, the results indicate that the choice of similarity measure might depend on whether the desired task is for design exploitation or exploration, as well as on the types of products in question. Table 8 qualitatively summarizes the differences in how the measures determine functional similarity.

Table 8. Description of how measures determine functional similarity and design-relevant considerations for use. Complexity refers to differences in the number of components and functions being implemented by the systems being compared. Weighting refers to if the measure can handle more information such as weighting of the importance of a function. Use in DbA refers to the potential for finding near or far stimuli based on the types of examples returned as most similar.

Measure	Similarity Principle	Complexity	Weighting	Use in DbA
SMC	Utilizes the <i>absence</i> of key function-flow pairs	Similar complexity	No	Near
DeltaCon	Utilizes the <i>absence</i> of key function-flow pairs	Similar complexity	Yes	Near
Jaccard	Matches function-flow pairs that exist in at least one of the systems	Similar complexity	Yes	Near
Cosine	Matches function-flow pairs that exist in at least one of the systems	Varying complexity	Yes	Near
NetSimile	Finds patterns in the <i>connections</i> between functions and flows with less focus on what the specific functions are	Varying complexity	Yes	Far
Spectral	Finds patterns in the <i>connections</i> between functions and flows with less focus on what the specific functions are	Similar complexity	Yes	Far

The energy harvesters represent a set of products of varying complexity that might not have

surface or form similarities, but are related to each other functionally. When performing design-by-analogy for such types of products, a computational approach to finding the similarity between them might be particularly useful. However, since it is possible to define the similarity between products in several ways, the measure choice can lead to different analogies that might influence a designer's trajectory. Therefore, the following are recommendations to consider:

- Use a measure like NetSimile or spectral distance for divergent design exploration
- Use a measure like NetSimile or spectral distance when searching for different or new methods to achieve specific functional implementations (e.g. a water-powered vs. wind-powered turbine or optical vs. mechanical switches in keyboards)
- Use a measure like cosine similarity when searching for examples of particular functional implementations for refinement or convergence

Finally, to truly understand the use of specific similarity measures in the context of design exploration or exploitation, it is important to determine whether the types of results provided by unexplored network-based distances, like the spectral distance, would be useful to designers in practice. Would the measures be retrieving examples that are “too far” or “just right”?

5 CONCLUSION

An empirical analysis of how different similarity measures determine the similarity of designs represented at a functional level indicated that the choice of measure will follow various constructs of similarity. As such, designers and scholars developing computational design-by-analogy design support tools must pay particular attention to the details of the similarity measure, and not simply rely on the notion that *any* metric can find “similar” designs. The analysis found some network measures, such as NetSimile and the spectral distance, to be a potentially viable alternative to vector measures for early-stage contexts, as they do not rely heavily on the lower-level features of the design representation (e.g. function-flow pairs in this work). However, what these measures consider to be functionally similar may not be immediately obvious or intuitive, and may be misleading during periods of convergence. The results are particularly relevant to determining near vs. far analogical stimuli and aiding in design exploitation vs. exploration. This research is a step toward understanding which similarity measures should be used during different design stages.

Though only tested on functional models in the present study, the results imply the need to carefully consider the choice of similarity metric in research that requires a measurement of design similarity, regardless of the design representation.

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REFERENCES

- [1] McAdams, D. A., Stone, R. B., and Wood, K. L., 1999. "Functional Interdependence and Product Similarity Based on Customer Needs". *Research in Engineering Design*, **11**(1), Apr., pp. 1–19.
- [2] Schuh, G., Riesener, M., and Rudolf, S., 2014. "Identifying Preferable Product Variants Using Similarity Analysis". *Procedia CIRP*, **20**, Jan., pp. 38–43.
- [3] Helmers, L., Horn, F., Biegler, F., Oppermann, T., and Müller, K.-R., 2019. "Automating the search for a patent's prior art with a full text similarity search". *PLOS ONE*, **14**(3), Mar., p. e0212103.
- [4] Chan, T. H., Mihm, J., and Sosa, M. E., 2017. "On Styles in Product Design: An Analysis of U.S. Design Patents". *Management Science*, **64**(3), pp. 983–1476.
- [5] Goel, A., 1997. "Design, analogy, and creativity". *IEEE Expert*, **12**(3), May, pp. 62–70.
- [6] Markman, A., Wood, K., Linsey, J., Murphy, J., and Laux, J., 2009. "Supporting Innovation by Promoting Analogical Reasoning". In *Tools for Innovation*. Oxford University Press, Inc., Nov., pp. 85–103.
- [7] Goucher-Lambert, K., Gyory, J. T., Kotovsky, K., and Cagan, J., 2020. "Adaptive Inspirational Design Stimuli: Using Design Output to Computationally Search for Stimuli that Impact

- Concept Generation”. *Journal of Mechanical Design*, **142**(9), pp. 1–37.
- [8] Goucher-Lambert, K., Moss, J., and Cagan, J., 2019. “A neuroimaging investigation of design ideation with and without inspirational stimuli—understanding the meaning of near and far stimuli”. *Design Studies*, **60**, Jan., pp. 1–38.
- [9] Kittur, A., Yu, L., Hope, T., Chan, J., Lifshitz-Assaf, H., Gilon, K., Ng, F., Kraut, R. E., and Shahaf, D., 2019. “Scaling up analogical innovation with crowds and AI”. *Proceedings of the National Academy of Sciences*, **116**(6), Feb., pp. 1870–1877.
- [10] Fu, K., Chan, J., Cagan, J., Kotovsky, K., Schunn, C., and Wood, K., 2013. “The Meaning of “Near” and “Far”: The Impact of Structuring Design Databases and the Effect of Distance of Analogy on Design Output”. *Journal of Mechanical Design*, **135**(2), Feb.
- [11] Murphy, J., Fu, K., Otto, K., Yang, M., Jensen, D., and Wood, K., 2014. “Function Based Design-by-Analogy: A Functional Vector Approach to Analogical Search”. *Journal of Mechanical Design*, **136**(10), Oct., p. 101102.
- [12] Linsey, J. S., Laux, J., Clauss, E. F., Wood, K. L., and Markman, A. B., 2007. “Effects of analogous product representation on design-by-analogy”. In *DS 42: Proceedings of ICED 2007, the 16th International Conference on Engineering Design*, J.-C. Bocquet, ed., Design Society.
- [13] Nagel, R. L., Midha, P. A., Tinsley, A., Stone, R. B., McAdams, D. A., and Shu, L. H., 2008. “Exploring the Use of Functional Models in Biomimetic Conceptual Design”. *Journal of Mechanical Design*, **130**(12), Dec., p. 121102.
- [14] McAdams, D. A., and Wood, K. L., 2002. “A quantitative similarity metric for design-by-analogy”. *Journal of Mechanical Design*, **124**(2), pp. 173–182.
- [15] Turner, C. J., and Linsey, J., 2016. “Analogies from Function, Flow and Performance Metrics”. In *Workshops Proceedings for the Twenty-fourth International Conference on Case-Based Reasoning (ICCBR 2016)*, A. Coman and S. Kapetanakis, eds. CEUR Workshop Proceedings, pp. 98–107.
- [16] Caldwell, B. W., and Mocko, G. M., 2011. “Functional Similarity at Varying Levels of Abstraction”. In *ASME 2010 International Design Engineering Technical Conferences and Computers*

and Information in Engineering Conference, Vol. 5: 22nd International Conference on Design Theory and Methodology; Special Conference on Mechanical Vibration and Noise. ASME, New York, Mar., pp. 431–441.

- [17] Weaver, J. M., Wood, K. L., Crawford, R. H., and Jensen, D., 2011. “Exploring Innovation Opportunities in Energy Harvesting Using Functional Modeling Approaches”. In *ASME 2011 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, Vol. 9: 23rd International Conference on Design Theory and Methodology; 16th Design for Manufacturing and the Life Cycle Conference. ASME, pp. 479–489.
- [18] Poppa, K. R., 2011. “Theory and application of vector space similarity measures in computer assisted conceptual design”. PhD thesis, School of Mechanical, Industrial, and Manufacturing Engineering, Corvallis, OR.
- [19] Chaudhari, A. M., 2019. “Similarity in Engineering Design : A Knowledge-Based Approach”. In *ASME 2019 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, Vol. 7: 31st International Conference on Design Theory and Methodology. ASME, New York, p. V007T06A045.
- [20] Anandan, S., Teegavarapu, S., and Summers, J. D., 2008. “Issues of Similarity in Engineering Design”. In *ASME 2006 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers Digital Collection, pp. 73–82.
- [21] Gill, A. S., Tsoka, A. N., and Sen, C., 2019. “Dimensions of Product Similarity in Design by Analogy: An Exploratory Study”. In *ASME 2019 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference*, American Society of Mechanical Engineers Digital Collection.
- [22] Ulrich, K. T., and Eppinger, S. D., 2004. *Product Design and Development*. McGraw-Hill/Irwin.
- [23] Hirtz, J., Stone, R., McAdams, D., Szykman, S., and Wood, K., 2002. “A Functional Basis for Engineering Design: Reconciling and Evolving Previous Efforts”. *Research in Engineering Design*, **13**(2), pp. 65–82.

- [24] Eisenbart, B., Gericke, K., and Blessing, L., 2013. “An analysis of functional modeling approaches across disciplines”. *AI EDAM*, **27**(3), Aug., pp. 281–289.
- [25] Hundal, M. S., 1990. “A Systematic method for developing function structures, solutions and concept variants”. *Mechanism and Machine Theory*, **25**(3), Jan., pp. 243–256.
- [26] Ullman, D. G., 2003. *The Mechanical Design Process*. McGraw-Hill.
- [27] Pahl, G., Beitz, W., Feldhusen, J., and Grote, K.-H., 2007. *Engineering Design: A Systematic Approach*, 3 ed. Springer-Verlag, London.
- [28] Stone, R. B., and Wood, K. L., 2000. “Development of a Functional Basis for Design”. *Journal of Mechanical Design*, **122**(4), Dec., pp. 359–370.
- [29] Little, A. D., and Wood, K. L., 1997. “Functional Analysis: A Fundamental Empirical Study for Reverse Engineering, Benchmarking, and Redesign”. In *ASME Design Theory and Methodology Conference*. ASME, pp. DETC97/DTM–3879.
- [30] Otto, K. N., and Wood, K. L., 1997. “Conceptual and Configuration Design of Products and Assemblies”. In *ASM International Handbook*, G. E. Dieter, ed., Vol. Volume 20, Materials Selection and Design. ASM International, ch. 1C, pp. 15–32.
- [31] Choi, S.-S., Cha, S.-H., and Tappert, C. C., 2010. “A Survey of Binary Similarity and Distance Measures”. *Journal of Systemics, Cybernetics and Informatics*, **8**(1), pp. 43–48.
- [32] Ahmed, F., and Fuge, M., 2018. “Creative Exploration Using Topic Based Bisociative Networks”. *arXiv:1801.10084 [cs]*, Jan.
- [33] Gyory, J. T., Goucher-Lambert, K., Kotovsky, K., and Cagan, J., 2019. “Exploring the Application of Network Analytics in Characterizing a Conceptual Design Space”. *Proceedings of the Design Society: International Conference on Engineering Design*, **1**(1), July, pp. 1953–1962.
- [34] Walsh, H. S., Dong, A., and Tumer, I. Y., 2018/ed. “The role of bridging nodes in behavioral network models of complex engineered systems”. *Design Science*, **4**.
- [35] Panyam, V., Huang, H., Pinte, B., Davis, K., and Layton, A., 2019. “Bio-Inspired Design for Robust Power Networks”. In 2019 IEEE Texas Power and Energy Conference (TPEC), pp. 1–6.
- [36] Li, Y., Wang, Z., Zhong, X., and Zou, F., 2019. “Identification of influential function modules

- within complex products and systems based on weighted and directed complex networks”. *Journal of Intelligent Manufacturing*, **30**(6), Aug., pp. 2375–2390.
- [37] Dong, A., 2017. “Functional lock-in and the problem of design transformation”. *Research in Engineering Design*, **28**(2), pp. 203–221.
- [38] Soundarajan, S., Eliassi-Rad, T., and Gallagher, B., 2014. “A Guide to Selecting a Network Similarity Method”. In Proceedings of the 2014 SIAM International Conference on Data Mining, Society for Industrial and Applied Mathematics, pp. 1037–1045.
- [39] Tantardini, M., Ieva, F., Tajoli, L., and Piccardi, C., 2019. “Comparing methods for comparing networks”. *Scientific Reports*, **9**(1), Nov., pp. 1–19.
- [40] Design Engineering Research Laboratory, 2020. The Design Repository. <http://fctest.mime.oregonstate.edu/repo/browse/>, December.
- [41] Wills, P., and Meyer, F. G., 2019. “Metrics for Graph Comparison: A Practitioner’s Guide”. *bioRxiv*, Apr., p. 611509.
- [42] Hagberg, A. A., Schult, D. A., and Swart, P. J., 2008. “Exploring Network Structure, Dynamics, and Function using NetworkX”. In *Proceedings of the 7th Python in Science Conference (SciPy 2008)*, G. Varoquaux, J. Millman, and T. Vaught, eds. SciPy Organizers, Pasadena, CA, pp. 11–16.
- [43] Berlingerio, M., Koutra, D., Eliassi-Rad, T., and Faloutsos, C., 2012. “NetSimile: A Scalable Approach to Size-Independent Network Similarity”. *arXiv:1209.2684 [physics, stat]*, Sept.
- [44] Koutra, D., Vogelstein, J. T., and Faloutsos, C., 2013. “DELTACON: A Principled Massive-Graph Similarity Function”. *arXiv:1304.4657 [physics]*, Apr.
- [45] Agyemang, M., Linsey, J., and Turner, C. J., 2017. “Transforming functional models to critical chain models via expert knowledge and automatic parsing rules for design analogy identification”. *AI EDAM*, **31**(4), Nov., pp. 501–511.
- [46] Nandy, A., Dong, A., and Goucher-Lambert, K., 2020. “A Comparison of Vector and Network-based Measures for Assessing Design Similarity”. In ASME 2020 International Design Engineering Technical Conferences and Computers and Information in Engineering Conference, American Society of Mechanical Engineers Digital Collection.

APPENDIX A CATEGORIZATION OF ENERGY HARVESTING DEVICES

Category	Systems	Unique Functions	Unique Flows	Total Pairs
Inductive	Perpetuum FSH/C	9	6	54
	Enocean Eco 100	11	7	77
	Clarkson U Prototype	13	7	91
	Michigan U PFIG	13	6	78
	U Texas Prototype	15	7	105
	Seiko Kinetic Watch	16	9	144
	AA Battery Harvester	15	9	135
	Socket	12	8	96
	Kinetic Flashlight	15	8	120
Piezoelectric	MIDE Volture	12	6	72
	Bistable Buckling Harvester	10	5	50
	Heel-impact Shoe Harvester	15	7	105
	Innowattech Road/Rail	12	3	36
	Piezo Backpack Straps	12	5	60
	U Texas Prototype	17	7	119
Wind	WindTamer	13	10	130
	Leviathan	12	9	108
	Enviro Energies	13	9	117
	Four Seasons	13	8	104
	Humdinger Wind Belt	9	8	72
	Michigan U Piezo Flag	10	8	80
Ocean-current/Wave	Nova Energy Tuna Turbine	13	10	130
	Columbia Power Manta Buoy	11	7	77
	Wing Wave Generator	10	8	80
Solar	Solar Heat Engine w/ Mirrors	16	7	112
	Tracking System	18	7	126
	Inflatable Mat	14	9	126
	Big Belly Trash Compactor	19	10	190
	Transparent Film on Window	8	6	48
	Seiko Solar Watch	16	8	128
Thermal	Seiko Thermic Watch	17	10	170
	Enocean ECT 310 Perpetuum	14	7	98
	Micropelt TE-power Probe	11	9	99
	Micropelt TE-Power Ring	11	8	88
	Micropelt STM-PEM	11	7	77
Hybrid	Solar Powered Sterling Engine	11	10	110
	Solar/Wind Streetlamp	15	10	150
	Kinesis Wind/Solar	17	11	187
	Hymini Wind/Solar Crank	18	12	216

*Additional data for consumer product dataset available on request

APPENDIX B LIST OF FUNCTIONS AND FLOWS

Functions	Flows
Separate	Solid
Distribute	Human
Import	Gas
Export	Liquid
Transfer	Mixture**
Guide	Human Energy
Couple	Mechanical Energy
Mix	- <i>Rotational Mechanical Energy</i>
Actuate	- <i>Translational Mechanical Energy</i>
Regulate	- <i>Vibrational Mechanical Energy</i>
Change	Pneumatic Energy
Stop	Hydraulic Energy
Convert	Light Energy
Store	Electrical Energy
Supply	Magnetic Energy*
Sense	Thermal Energy
Indicate	Acoustic Energy**
Process*	Chemical Energy**
Stabilize	Status
Secure	Control
Position	

*Only in energy harvesting dataset **Only in consumer products dataset

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